

# Mathematica 11.3 Integration Test Results

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Test results for the 9 problems in "4.7.4 x^m (a+b trig^n)^p.m"

Problem 1: Result more than twice size of optimal antiderivative.

$$\int \frac{x}{a + b \sin[x]^2} dx$$

Optimal (type 4, 203 leaves, 9 steps):

$$\begin{aligned} & -\frac{i x \operatorname{Log}\left[1 - \frac{b e^{2 i x}}{2 a+b-2 \sqrt{a} \sqrt{a+b}}\right]}{2 \sqrt{a} \sqrt{a+b}} + \frac{i x \operatorname{Log}\left[1 - \frac{b e^{2 i x}}{2 a+b+2 \sqrt{a} \sqrt{a+b}}\right]}{2 \sqrt{a} \sqrt{a+b}} - \\ & \frac{\operatorname{PolyLog}\left[2, \frac{b e^{2 i x}}{2 a+b-2 \sqrt{a} \sqrt{a+b}}\right]}{4 \sqrt{a} \sqrt{a+b}} + \frac{\operatorname{PolyLog}\left[2, \frac{b e^{2 i x}}{2 a+b+2 \sqrt{a} \sqrt{a+b}}\right]}{4 \sqrt{a} \sqrt{a+b}} \end{aligned}$$

Result (type 4, 545 leaves):

$$\begin{aligned}
 & \frac{1}{4 \sqrt{-a(a+b)}} \left( 4 x \operatorname{ArcTanh} \left[ \frac{a \operatorname{Cot}[x]}{\sqrt{-a(a+b)}} \right] - 2 \operatorname{ArcCos} \left[ 1 + \frac{2a}{b} \right] \operatorname{ArcTanh} \left[ \frac{\sqrt{-a(a+b)} \operatorname{Tan}[x]}{a} \right] \right) + \\
 & \left( \operatorname{ArcCos} \left[ 1 + \frac{2a}{b} \right] - 2 i \operatorname{ArcTanh} \left[ \frac{a \operatorname{Cot}[x]}{\sqrt{-a(a+b)}} \right] + 2 i \operatorname{ArcTanh} \left[ \frac{\sqrt{-a(a+b)} \operatorname{Tan}[x]}{a} \right] \right) \\
 & \operatorname{Log} \left[ \frac{\sqrt{2} \sqrt{-a(a+b)} e^{-i x}}{\sqrt{-b} \sqrt{2a+b-b \operatorname{Cos}[2x]}} \right] + \\
 & \left( \operatorname{ArcCos} \left[ 1 + \frac{2a}{b} \right] + 2 i \left( \operatorname{ArcTanh} \left[ \frac{a \operatorname{Cot}[x]}{\sqrt{-a(a+b)}} \right] - \operatorname{ArcTanh} \left[ \frac{\sqrt{-a(a+b)} \operatorname{Tan}[x]}{a} \right] \right) \right) \\
 & \operatorname{Log} \left[ \frac{\sqrt{2} \sqrt{-a(a+b)} e^{i x}}{\sqrt{-b} \sqrt{2a+b-b \operatorname{Cos}[2x]}} \right] - \left( \operatorname{ArcCos} \left[ 1 + \frac{2a}{b} \right] + 2 i \operatorname{ArcTanh} \left[ \frac{\sqrt{-a(a+b)} \operatorname{Tan}[x]}{a} \right] \right) \\
 & \operatorname{Log} \left[ \frac{2a(a+b-i\sqrt{-a(a+b)}) (1-i \operatorname{Tan}[x])}{b(a+\sqrt{-a(a+b)} \operatorname{Tan}[x])} \right] - \\
 & \left( \operatorname{ArcCos} \left[ 1 + \frac{2a}{b} \right] - 2 i \operatorname{ArcTanh} \left[ \frac{\sqrt{-a(a+b)} \operatorname{Tan}[x]}{a} \right] \right) \\
 & \operatorname{Log} \left[ \frac{2a(a+b+i\sqrt{-a(a+b)}) (1+i \operatorname{Tan}[x])}{b(a+\sqrt{-a(a+b)} \operatorname{Tan}[x])} \right] + \\
 & i \left( \operatorname{PolyLog} \left[ 2, \frac{(2a+b-2i\sqrt{-a(a+b)}) (-a+\sqrt{-a(a+b)} \operatorname{Tan}[x])}{b(a+\sqrt{-a(a+b)} \operatorname{Tan}[x])} \right] - \right. \\
 & \left. \operatorname{PolyLog} \left[ 2, \frac{(2a+b+2i\sqrt{-a(a+b)}) (-a+\sqrt{-a(a+b)} \operatorname{Tan}[x])}{b(a+\sqrt{-a(a+b)} \operatorname{Tan}[x])} \right] \right)
 \end{aligned}$$

**Problem 6: Result more than twice size of optimal antiderivative.**

$$\int \frac{x}{a+b \operatorname{Cos}[x]^2} dx$$

Optimal (type 4, 203 leaves, 9 steps):

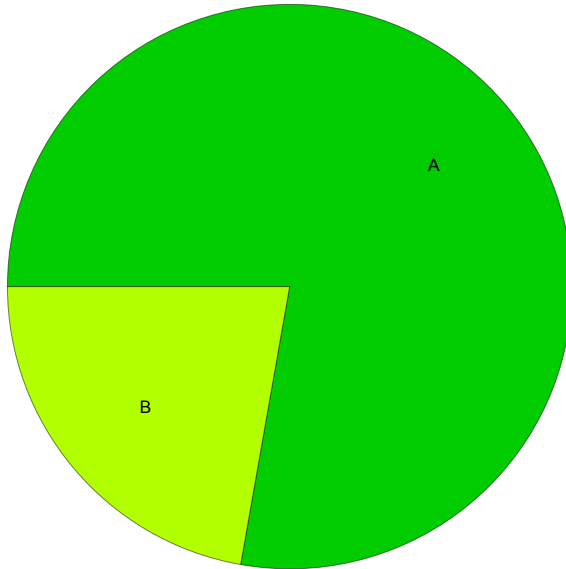
$$\begin{aligned}
 & - \frac{i x \operatorname{Log}\left[1 + \frac{b e^{2ix}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right]}{2\sqrt{a}\sqrt{a+b}} + \frac{i x \operatorname{Log}\left[1 + \frac{b e^{2ix}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right]}{2\sqrt{a}\sqrt{a+b}} - \\
 & \frac{\operatorname{PolyLog}\left[2, -\frac{b e^{2ix}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right]}{4\sqrt{a}\sqrt{a+b}} + \frac{\operatorname{PolyLog}\left[2, -\frac{b e^{2ix}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right]}{4\sqrt{a}\sqrt{a+b}}
 \end{aligned}$$

Result (type 4, 532 leaves):

$$\begin{aligned}
 & \frac{1}{4\sqrt{-a(a+b)}} \left( 4x \operatorname{ArcTanh}\left[\frac{(a+b)\operatorname{Cot}[x]}{\sqrt{-a(a+b)}}\right] + 2 \operatorname{ArcCos}\left[-1 - \frac{2a}{b}\right] \operatorname{ArcTanh}\left[\frac{a \operatorname{Tan}[x]}{\sqrt{-a(a+b)}}\right] + \right. \\
 & \left. \left( \operatorname{ArcCos}\left[-1 - \frac{2a}{b}\right] - 2i \left( \operatorname{ArcTanh}\left[\frac{(a+b)\operatorname{Cot}[x]}{\sqrt{-a(a+b)}}\right] + \operatorname{ArcTanh}\left[\frac{a \operatorname{Tan}[x]}{\sqrt{-a(a+b)}}\right] \right) \right) \right) \\
 & \operatorname{Log}\left[\frac{\sqrt{2}\sqrt{-a(a+b)} e^{-ix}}{\sqrt{b}\sqrt{2a+b+b\cos[2x]}}\right] + \\
 & \left( \operatorname{ArcCos}\left[-1 - \frac{2a}{b}\right] + 2i \left( \operatorname{ArcTanh}\left[\frac{(a+b)\operatorname{Cot}[x]}{\sqrt{-a(a+b)}}\right] + \operatorname{ArcTanh}\left[\frac{a \operatorname{Tan}[x]}{\sqrt{-a(a+b)}}\right] \right) \right) \\
 & \operatorname{Log}\left[\frac{\sqrt{2}\sqrt{-a(a+b)} e^{ix}}{\sqrt{b}\sqrt{2a+b+b\cos[2x]}}\right] - \left( \operatorname{ArcCos}\left[-1 - \frac{2a}{b}\right] + 2i \operatorname{ArcTanh}\left[\frac{a \operatorname{Tan}[x]}{\sqrt{-a(a+b)}}\right] \right) \\
 & \operatorname{Log}\left[\frac{2(a+b)\left(-ia + \sqrt{-a(a+b)}\right)\left(-i + \operatorname{Tan}[x]\right)}{b\left(a+b + \sqrt{-a(a+b)}\operatorname{Tan}[x]\right)}\right] - \\
 & \left( \operatorname{ArcCos}\left[-1 - \frac{2a}{b}\right] - 2i \operatorname{ArcTanh}\left[\frac{a \operatorname{Tan}[x]}{\sqrt{-a(a+b)}}\right] \right) \\
 & \operatorname{Log}\left[\frac{2(a+b)\left(ia + \sqrt{-a(a+b)}\right)\left(i + \operatorname{Tan}[x]\right)}{b\left(a+b + \sqrt{-a(a+b)}\operatorname{Tan}[x]\right)}\right] + \\
 & i \left( \operatorname{PolyLog}\left[2, \frac{\left(2a+b-2i\sqrt{-a(a+b)}\right)\left(a+b - \sqrt{-a(a+b)}\operatorname{Tan}[x]\right)}{b\left(a+b + \sqrt{-a(a+b)}\operatorname{Tan}[x]\right)}\right] - \right. \\
 & \left. \operatorname{PolyLog}\left[2, \frac{\left(2a+b+2i\sqrt{-a(a+b)}\right)\left(a+b - \sqrt{-a(a+b)}\operatorname{Tan}[x]\right)}{b\left(a+b + \sqrt{-a(a+b)}\operatorname{Tan}[x]\right)}\right] \right)
 \end{aligned}$$

## Summary of Integration Test Results

9 integration problems



A - 7 optimal antiderivatives

B - 2 more than twice size of optimal antiderivatives

C - 0 unnecessarily complex antiderivatives

D - 0 unable to integrate problems

E - 0 integration timeouts